REVERSE MORTGAGES FOR THE LOW-INCOME ELDERLY: AN ECONOMIC ANALYSIS

By

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An important question concerning the reverse annuity mortgage (RAM) is its potential for impacting the poverty problem. However, before directly addressing the poverty question, the economics of RAMs for the elderly must be examined. The literature contains calculations for the rates of return to the lender for the RAM and its variations, but no calculations for the rate of return to the elderly homeowner. In addition, once the rate of return to the homeowner is found, a method is needed to determine whether the return is adequate compared to investment alternatives.

This study develops cash flow models to find the internal rates of return in the following situations for the low-income elderly household: homeowner without a reverse mortgage; homeowner with a standard RAM; homeowner with a RAM which includes a premium for life tenure; and a homeowner with a reverse shared-appreciation mortgage (RSAM). A new application of portfolio theory is then used to complete the model by providing a yardstick for comparison with alternative investments.

An application of each model is made using data for the elderly in the Orlando Standard Metropolitan Statistical Area (SMSA). Two results from the Orlando Examples are as follows: Homeowners do receive an adequate return on home equity compared to investment alternatives; return on home equity is greater for a RAM with a premium for life tenure than with a standard RAM.

A self-subsidizing program is developed for the RSAM with the objective that the benefit be no less than the mean deficit below poverty. Households with high home values pay premiums in order to subsidize households with low home values. In the Orlando example, the objective can be met for 42% of the households without subsidies, and an additional 20% with subsidies. A subsidy from the government is required to meet the objective for the remaining 38%.

CHAPTER I

INTRODUCTION

Elderly homeowners with incomes below the poverty line have, on average, very little in assets other than the equity in their homes. For this reason, an instrument has been developed to convert home equity into income. The reverse mortgage, also known as a reverse annuity mortgage (RAM), involves payments from a lender to a household for a fixed term. Each payment is a loan, and interest charges are added to the rising loan balance. The payments are calculated to result in a loan balance at the end of the fixed term which is in a predetermined ratio with the value at the time of origination. (There are other variations of this instrument.)

An important question concerning the RAM is its potential for impacting the poverty problem. However, there is another issue to address before directly assaulting the poverty question. Doubts remain as to the adequacy of economic returns to the homeowner participating in the RAM [1]. Weinrobe, Guttentag, and Garnett calculate the rate of return to the lender for the RAM and its variations [2-5]. The literature does not contain calculations of the rate of return to the homeowner. In addition, if a rate of return to the homeowner can be found, a method is needed to determine

whether the return is adequate compared to investment alternatives.

Chapter II addresses the question of the economic return to the homeowner. Possession and use of a home does have value (imputed rent), and an income property pro forma is used as a guide to find an internal rate of return on homeowner equity. A new application of portfolio theory is then used to complete the model by providing a yardstick for comparison with alternative investments.

Chapter III extends the model to include a household participating in a standard RAM and a RAM with a premium for life tenure. The possibility of a household surviving past the fixed term of a RAM results in risks for both the lender and the borrower. The lender is concerned with default risk, while the borrower is concerned with a forced sale to pay off the loan balance. An early work by Guttentag presents one method for incorporating mortality distributions into the calculations for the lender's return [3]. Weinrobe, in a study done concurrently with the work herein, developed insurance premiums for RAMs [6]. The model in Chapter III addresses the return on home equity, most importantly the result of adding a premium for life tenure.

Chapter IV deals with the issue of the impact on poverty. For this reason, attention is turned to a newer version of the reverse mortgage which results in higher payments to the household than is possible with the RAM. It is the reverse shared-appreciation mortgage (RSAM), which gives the

household a lower interest rate (resulting in higher payments) in exchange for a share in the property's appreciation [5]. The RSAM also features life tenure.

For the purpose of this study, impact on poverty is measured by the proportion of elderly households receiving RSAM payments equal to or greater than the mean deficit below poverty. Reverse shared-appreciation mortgage payments to households vary because of the range of property values among the elderly. The approach used here is to charge premiums on households with high-value homes in order to subsidize households with low-value homes. The model is extended to calculate the maximum premium that the household is willing to pay.

Data on low-income elderly households in the Orlando Standard Metropolitan Statistical Area (SMSA) are used as an application of the models in Chapters II, III, and IV. In Chapters III and IV, RAM payments and RSAM payments are made from age 65 to life expectancy. The Orlando application serves as an example only; it is not suggested that the results have general application.

CHAPTER II

APPLICATION OF PORTFOLIO THEORY TO RETURN ON HOME EQUITY

The investment portfolio of the low-income elderly consists almost entirely of home equity. The U.S. Department of Housing and Urban Development conducted a survey of 1,575 elderly homeowners in 1975. Average home equity for the sample was $25,630 with an average of $18,700 in other assets. The homeowners with incomes below the poverty line averaged $3,698 in other assets [7].

The importance of home equity to the low-income elderly creates a demand for a method that can measure the return on home equity for comparison with other investment alternatives. Such a method would be useful in retirement counseling. For example, a counselor could show that a homeowner is not receiving an adequate return on home equity, and show the advantages of relocating to an area which offers less expensive housing, lower taxes, or reduced living expenses.

The same method could also be used for comparing similar households in different states or in different Standard Metropolitan Statistical Areas (SMSAs). For example, investors could identify areas that are not economically favorable for elderly homeowners and create a more attractive alternative. If the alternative involves relocating, the

investors could serve an entire spectrum of needs, including the disposal of property, moving, and providing new investments and living accommodations which meet the special requirements of the elderly.

The Efficient Frontier

Using portfolio theory, this paper will compare the return on home equity with the return on investment alternatives. In portfolio theory, the efficient frontier of investments is defined by two points in risk-return space and is normally depicted as shown in Figure 2-1 [8].

The vertical axis represents the expected value of return or E(r). The horizontal axis represents the risk involved in the return, assumed to be the standard deviation, S(r). The point $(0, r_f)$ on the vertical axis is the return available on a risk-free asset. The point $(S(r_p), E(r_p))$ represents the risk and return of a portfolio of risky securities. (When the portfolio under consideration is the market portfolio of all risky securities, this line is referred to as the capital market line or CML.)

An investor chooses an investment portfolio based on his or her attitude toward risk. An investor unwilling to accept any risk would invest only in the risk-free asset. A risk-seeker would invest only in the portfolio of risky securities and, in some cases, will borrow funds in order to invest more. A typical investor generally is considered to be risk averse. Risk averse, in this sense, means that the investor is willing to accept some risk but demands a higher return as the risk increases.

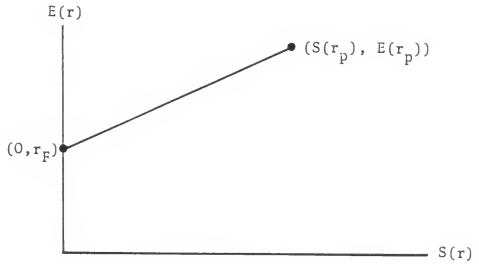

Figure 2-1

Efficient Frontier of Investments

A utility function is sometimes used to represent investor attitude toward risk. Figure 2-2 adds a possible utility function to the efficient frontier shown in Figure 2-1. Each curve represents a solution to the utility function where utility (satisfaction) is constant. Stated differently, each combination of risk and return on a utility curve gives the investor the same level of satisfaction. The arrow in Figure 2-2 indicates the direction of increasing utility. Point A in Figure 2-2 represents an investment portfolio where 100% of the funds are invested in the risk-free asset; at point C, 100% of the funds are invested in the risky portfolio. The investor who is risk averse, with the utility curves shown, would choose proportions of the risk-free asset and risky portfolio at point B, where the highest utility curve is intersected for the greatest satisfaction. (Investing to the right of point C involves negative amounts of the risk-free asset, which means borrowing at the risk-free rate.)

The efficient frontier represents the investment alternatives; in order to investigate the reinvestment decision, one must view homeownership as an investment and find a risk-return relationship. If the point representing the risk-return relationship falls on or above the efficient frontier, the household should remain in the home. A point falling below the efficient frontier is a signal to sell and reinvest.

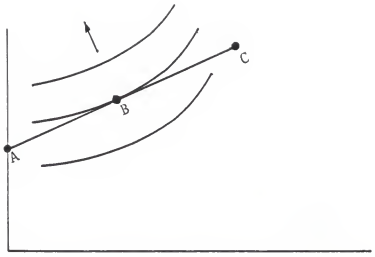

Figure 2.2
Efficient Frontier with Utility Curves

Homeownership as an Investment

It is useful to view the household's situation with a standard income property pro forma.

Potential Gross Income
*less vacancy and collection losses

Effective Gross Income
less property taxes
less insurance, repairs, and maintenance
*less management cost

Net Operating Income
*less debt service

Before Tax Cash Flow
*less taxes

After Tax Cash Flow

The items with an asterisk will be set to zero for low-income households. Vacancy and collection losses are zero because the owners are occupying their own home. There are no management costs because they perform that function. Only households without mortgages are being considered here; therefore, debt service is zero. Gross income for the household is imputed rent--imputed rent is never taxed; therefore, taxes are also zero.

With the above adjustments, the pro forma for an elderly household is reduced to the following.

Imputed Rent
less property taxes
less insurance, repairs, and maintenance

After Tax Cash Flow

Note that with no mortgage and no taxes, net operating income,

before tax cash flow, and after tax cash flow are all the same. It follows that the overall capitalization rate, the before tax equity dividend rate, and the after tax equity dividend rate are all equal. Net operating income (NOI) and the overall capitalization rate (R) will be used in the equations that follow. R = NOI/investment.

The pro forma developed above can be used for a cash flow model; the model adopts the additional assumption that NOI and property value increase with inflation. The present value factor for annuities changing in constant ratio is given by

$$PVFA = \frac{1 - (1+X)^N / (1+I)^N}{I - X} \qquad (1)$$

where PVFA = present value factor for the annuity

X = constant ratio change in income

N = number of years

I = rate of interest/yield

In this case, X will be equal to the rate of inflation.[1] By definition, I will be equal to the internal rate of return (IRR) when net present value equals zero. The present value factor for reversion is $1 / (1+I)^N$. The present value factor for a home that has its value increasing with inflation becomes

$$PVFR = (1+X)^N / (1+I)^N \qquad (2)$$

where PVFR = present value factor for reversion with inflation

X, N, I defined as before

Employing the factors from above, the expression for net present value (NPV) is

$$NPV = [PVFA \times NOI] + [PVFR \times H] - H \quad (3)$$

where H is the value of the home at the beginning of the holding period. Substituting equations (1) and (2) into equation (3) yields

$$NPV = [NOI \times \frac{1 - (1+X)^N / (1+I)^N}{I - X}] + [H \times (1+X)^N / (1+I)^N] - H$$

By definition, NPV = 0 when I = IRR, and NOI = R x H, therefore

$$NPV = 0 = [R \times H \times \frac{1 - (1+X)^N / (1+IRR)^N}{IRR - X}] + [H \times (1+X)^N / (1+IRR)^N] - H$$

Dividing through by H and substituting $K = (1+X)^N / (1+IRR)^N$ yields

$$0 = R \times \frac{(1-K)}{IRR-X} + K - 1 \text{ then}$$

$$0 = [R \times (1-K)] + [(K-1) \times (IRR-X)] \quad (4)$$

The equality in equation (4) holds if IRR = R+X. Also note that the equality holds for any value of K; it follows, then, that the equality holds for any value of N.

To summarize the algebra from above, it shows that the return on home equity is reduced to a very simple relationship, namely

$$IRR = R + \text{inflation rate} \quad (5)$$

In addition, IRR does not depend on the value of N; therefore, life expectancy is not a factor. The assumptions required in order for equation (5) to hold are no mortgage; no income tax; and NOI and home value increasing with inflation. Once the overall capitalization rate is computed, the expected

value of IRR depends entirely upon expectations of inflation.

Internal rate of return by itself is not adequate for comparison with alternative investments; a measure of risk is required. A standard deviation for IRR can be obtained by expressing inflation expectations as a probability distribution.

Transaction costs are normally added to a model once the relationships are found subject to the assumption of a perfect capital market.[2] The reversion term in equation (3) becomes PVFR x F x H where F is a factor for a sales commission. For example, F = .94 for a 6% sales commission. The last term in equation (3), the amount of the investment, must also be adjusted. The homeowner is locked into the investment and, in order to reinvest the funds, must pay a sales commission. Therefore, the last term in equation (3) becomes F x H; the amount of the investment equals the proceeds of sale after the sales commission is deducted. Equation (3), modified for transaction costs, becomes

$$NPV = [PVFA \times NOI] + [PVFR \times F \times H] - [F \times H] \quad (6)$$

Performing the same substitutions and operations as before, the equivalent to equation (4) is

$$0 = [R \times (1-K)] + [(K-1) \times F \times (IRR-X)] \quad (7)$$

The equality in equation (7) holds if IRR = R/F + X. As before, the equality holds for any value of K; it follows, again, that the equality holds for any value of N. The introduction of transaction costs results in a simple

relationship for return on home equity, where

$$IRR = \frac{R}{F} + \text{inflation rate} \qquad (8)$$

does not vary with life expectancy.

Application

Return on Home Equity

This section uses the model developed in the previous section to calculate the return on home equity for the low-income elderly in the Orlando SMSA. The state of Florida is chosen for this application because a $25,000 homestead exemption largely eliminates the burden of property tax (59% of homeowners with homes valued under $30,000 reported real estate taxes under $100). The group under consideration is 65 or older and is in the income range from $3,000 to $6,999; this income range includes the poverty line for both individuals and couples 65 and over.

The Annual Housing Survey for the Orlando SMSA provides figures necessary for the calculation of an overall capitalization rate. (The survey was conducted in 1981 and the results published in 1984.) The survey found that median home value is $38,300 and that mean real estate taxes are $6 per $1,000 value. The taxes for the median home equate to a rate of $17.28 per $1,000 on the amount in excess of the $25,000 homestead exemption. Median housing costs are $91 per month or $1,092 per year. According to a U.S. Department of Labor publication, insurance, repairs, and maintenance account for 16.9% of shelter costs.[3] The resulting expenses for the

elderly household are

Property taxes ($38,300-25,000) x $\frac{\$17.28}{\$1,000}$ = $230

Insurance, repairs, and maintenance $1,092x16.9% = $185

Imputed rent is derived from the median home value of $38,300--an annual gross income multiplier of 10 yields an annual rent of $3,830.[4] The resulting pro forma is

$3,830	Imputed rent
- 230	Property taxes
- 185	Insurance, repairs, and maintenance
$3,415	NOI

NOI divided by the median home value gives an overall capitalization rate.

R = NOI/H = $3,415/$38,300 = .0892

From equation (5), with no transaction costs

IRR = R + inflation rate = .0892 + inflation rate

A standard deviation for IRR is required as a measure of risk. Two probability distributions for inflation are illustrated below. Both distributions assume that the most likely inflation rate is 4%, but distribution B shows less certainty about the mean. The expected values and standard deviations for IRR are as shown.

Inflation	IRR	Probability Distribution A	Probability Distribution B
2%	10.92%	10%	20%
4%	12.92%	70%	50%
6%	14.92%	20%	30%
Expected value (IRR)		13.12%	13.12%
Standard deviation (IRR)		.0108	.014

The values for risk and return can now be plotted for comparison with investment alternatives.

Risk-Return Space for the Elderly Household

It is now necessary to attach values to the two points defining the efficient frontier. The yield on a long-term Treasury Bond is employed as a proxy for the yield on a risk-free asset.[5] The yield on Treasury Bonds maturing in 30 years was 7.5% on May 23, 1986.

The index for the Standard and Poor 500 is used as a proxy to obtain a yield for a risky portfolio. Dividend income is included as well as capital gains. The return on a risky portfolio can vary with holding strategy; therefore, holding periods from three years to six years are tested. Within each holding strategy, an expected return is calculated assigning equal probabilities to the five most recent periods. (The last calendar year included is 1985.) Standard deviations are also calculated. The results are shown in Table 2-1.

Figure 2-3 shows the efficient frontier, developed in this section. If our assumptions for the perfect capital market include perfect information for transactors, the line connecting the point for a 3 year holding period would be chosen, and the others ignored.

The point representing the elderly household depends on the household's expectations of inflation. Point A on the graph uses the IRRs from the previous section, which vary with inflation, and distribution A. Point B similarly is derived with distribution B. If the truth about inflation expectations lies somewhere between the points representing

Table 2-1

Return on Portfolio of Standard and Poor's 500 Stocks
(No Transaction Fees, No Income Tax)

Holding Period	Returns for the Five Periods					E(R)	S(R)
	1979-81	80-82	81-83	82-84	83-85		
3 Years	.1458	.1492	.1152	.1625	.1928	.1531	.0252
	1978-81	79-82	80-83	81-84	82-85		
4 Years	.1235	.1589	.1660	.1023	.1957	.1493	.0329
	1977-81	78-82	79-83	80-84	81-85		
5 Years	.0777	.1373	.1702	.1461	.1380	.1339	.0305
	1976-81	77-82	78-83	79-84	80-85		
6 Years	1.057	.0954	.1497	.1535	.1693	.1347	.0288

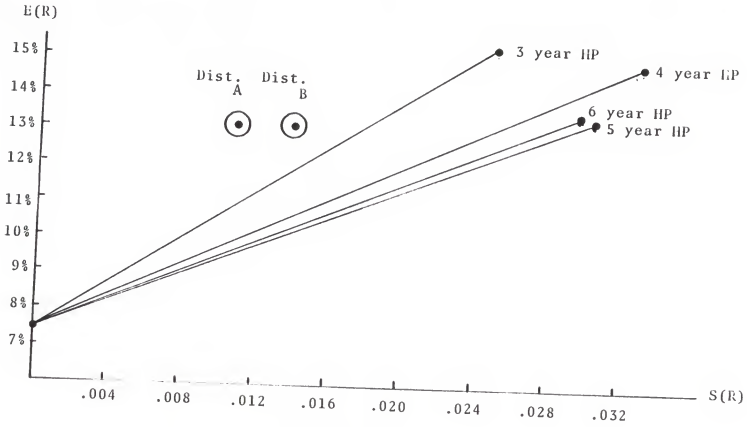

Figure 2-3

Efficient Frontier With No Transaction Costs

distributions A and B, the low-income elderly in the Orlando SMSA are, on average, receiving an adequate return on home equity. The next section will test the effects of transaction costs on these relationships.

Introduction of Transaction Costs

Two types of transaction costs will be included in the analysis. The first is a 6% sales commission on the sale of the home. The second type is a broker's fee on transactions of the stock market portfolio. A fee of .85% of the transaction is charged when buying, as well as a fee of .85% of the transaction when selling. (The fees are representative of discount broker rates.) Table 2-2 shows the results for the risky portfolio with transaction costs included.

The next logical step is to include income taxes, which would reduce the return on the risk-free asset as well as on the risky portfolio. However, the poverty line for 1980 is under $4,000 for an individual over 65, and Social Security benefits are not taxed. At that time, the regular exemption, the over 65 exemption, and the zero bracket amount totaled $4,300. Similarly, a couple both 65 or over can have an income of $7,400, in addition to Social Security, without being taxed. The poverty line for couples over 65 is under $5,000. Therefore, income tax is not considered a factor for elderly households at or below the poverty line.

Internal rates of return are calculated with the use of equation (8); the 6% sales commission results in F = .94.

Table 2-2

Return on Portfolio of Standard and Poor's 500 Stocks--
Transaction Fees Included (No Income Tax)

Holding Period	Returns for the Five Periods					E(R)	S(R)
	1979-81	80-82	81-83	82-84	83-85		
3 Years	.1395	.1429	.1091	.1561	.1862	.1467	.0250
	1978-81	79-82	80-83	81-84	82-85		
4 Years	.1189	.1541	.1611	.0977	.1907	.1445	.0327
	1977-81	78-82	79-83	80-84	81-85		
5 Years	.0742	.1337	.1663	.1423	.1342	.1301	.0304
	1976-81	77-82	78-83	79-84	80-85		
6 Years	.1026	.0924	.1465	.1502	.1660	.1315	.0287

$$IRR = \frac{R}{F} + \text{inflation rate}$$

$$= \frac{.0892}{.94} + \text{inflation rate}$$

$$= .0949 + \text{inflation rate}$$

In this case, the 6% sales commission results in a requirement that alternative investments have a return .57% (.0949-.0892) higher in order to attract reinvestment of home equity.

Figure 2-4 illustrates the results. The same two probability distributions for inflation are used, but the points for the lower-yielding risky portfolios are omitted. Transaction costs do not alter the relationships; the elderly in the Orlando SMSA receive, on average, an adequate return on home equity.

The extra line labeled "October" in Figure 2-4 deserves comment. The Treasury Bond yield in October was 10.6%, compared to 7.5% in May. Assuming an inflation rate of 4%, the real rate in May was near 3.5%, while the real rate in October was at an abnormally high level of about 6.6%. It appears that Treasury Bond yields in October were at a point in a cycle where yields were lagging behind the drop in inflation. Therefore, the difference in the May and October lines are due to cycles. The October line is inappropriate for a long-term investment analysis because it incorporates real rates which are abnormally high. On the other hand, if the line moves up because of an increase in inflation, the household's expectations of inflation will increase correspondingly, and the relationships will remain essentially unchanged.

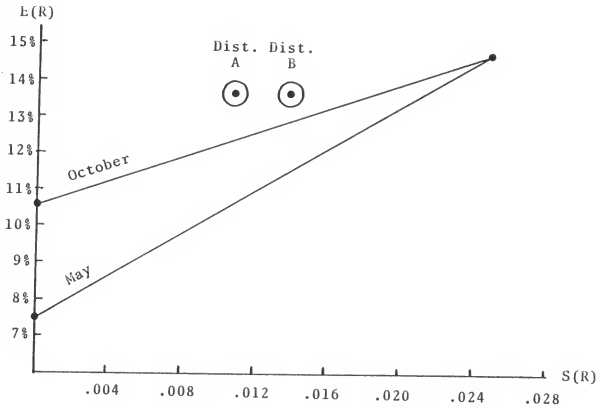

Figure 2-4

Efficient Frontier With Transaction Costs

Conclusion

An adequate return on home equity is important to the low-income elderly because, on average, they have very little in other assets. This paper shows a simple method which can be used to calculate the return on home equity, as well as a measure for risk. The only values required for the method are an overall capitalization rate; a probability distribution for expectations of inflation; and, if applicable, a sales commission.

Portfolio theory is applied to determine whether the return on home equity is adequate compared to investment alternatives. This procedure requires a proxy for a risk-free asset and a proxy for a portfolio of risky securities. The two proxies define the efficient frontier of investments, which is the yardstick for comparison with the return on home equity.

Application of the model using the data from the Orlando SMSA shows that the elderly homeowners there do, on average, receive an adequate return on home equity. This was true both with and without transaction costs.

Notes

1. For some applications, it might be more useful to define X as the rate of appreciation and let appreciation = inflation + delta. The delta could allow for regional or neighborhood differences and be either positive or negative.

2. "There are many transactors in the capital market, and no single transactor is large enough to affect prices or interest rates. . . . Every market participant has the same information about market prices and the relevant terms of any

transaction. Transactions in the capital market can be made without payment of charges other than the ruling market rate of interest" [8, p. 7].

3. The U.S. Department of Labor, Bureau of Labor Statistics, News: Three Budgets for a Retired Couple, Autumn 1978, release of August 20, 1979, shows shelter costs for homeowners in Orlando to be $1,066 in the lower budget--this agrees closely to the $1.092 figure extracted from the Annual Housing Survey. Bureau of Labor Statistics Bulletin 1570-6, which explains the methods of calculation for the three budgets, is the source of the 16.9% figure.

4. The normal range for the multiplier is between 4 and 10 [9]. The use of 10 is conservative for this application, keeping the R low.

5. This choice is based on the long-term nature of the investment decision and the need to be consistent with the 30-year mortgage rate to be added to the model in the next chapter.

CHAPTER III

IMPACT OF REVERSE MORTGAGES ON RETURN ON HOME EQUITY

This paper examines the economic returns of reverse mortgages to low-income elderly homeowners. Previous works look at the economic returns to lenders [2-5]. The rate of return to the lender can be compared to the market return on mortgage loans which have similar risk factors. A new application of portfolio theory is used here to compare the return on home equity to the return on investment alternatives.

The reverse mortgage, also known as a reverse annuity mortgage (RAM), involves payments from a lender to a household for a fixed term. The annuity payments plus interest result in a loan balance that grows to 80% of the initial home value at the end of the fixed term.[1] The reverse mortgages examined here are for males, females, and couples who receive benefits from age 65 to life expectancy.

One assumption of the perfect capital markets is that there are no charges for transactions other than the ruling market rate of interest. Therefore, an initial analysis is made with no transaction costs, followed by an analysis with transaction costs included to test the effect on the relationships. (Other perfect market assumptions are also invoked: "there are many transactors in the capital market,

and no single transactor is large enough to affect prices or interest rates . . . every market participant has the same information about market prices and the relevant terms of any transaction" [8, p. 7].)

Both lenders and households face certain risks by participating in reverse mortgages. Lenders are concerned with the default risk posed by households surviving past life expectancy. Households are concerned by the risk of being forced to sell their homes in order to pay off the loans. Therefore, premiums to offset these risks are also investigated.

Characteristics of a Reverse Mortgage Viewed to Life Expectancy

This section will formulate a cash flow model to compare the internal rate of return for a household without a reverse mortgage (homeowner IRR) with the internal rate of return for a household participating in a reverse mortgage (RM IRR). This example is based on the simplifying assumption that all individuals live to life expectancy.

The pro forma for a home investment, with a number of items set to zero for low-income elderly households without a mortgage, reduces to the following.[2]

Imputed Rent
less property taxes
less insurance, repairs, and maintenance
Net Operating Income (NOI)

With no mortgage and no tax on imputed rent, before-tax cash flow and after-tax cash flow are equal to NOI. The expression for net present value (NPV) is

$$NPV = [PVFA \times NOI] + [PVFR \times H \times (1+X)^N] - H \quad (1)$$

where PVFA = present value factor for annuities changing in constant ratio[3]

PVFR = present value factor for reversion

H = value of home at age 65 (start of holding period)

X = inflation rate

N = number of years

In Chapter II, it is shown that if NOI and H increase with inflation, equation (1) is satisfied (NPV = 0) when

$$\text{homeowner IRR} = R + \text{inflation rate} \quad (2)$$

where R = overall capitalization rate = NOI/H

Equation (2) holds for any value of N.

In order to find RM IRR, the reverse mortgage annuity (RM annuity) and satisfaction of the loan balance must be added to the NPV relationship (loan balance = 80% of H).

$$NPV = [PVFA \times NOI] + [PVFCA \times \text{RM annuity}] + PVFR \times [(H \times (1+X)^N) - (.8 \times H)] - H \quad (3)$$

where PVFCA = present value factor for a constant annuity

The IRR which satisfies equation (3) is found by standard iterative procedures.

Numerical values derived from the most recent Annual Housing Survey for the Orlando Standard Metropolitan Statistical Area (SMSA) are used as an application of the above equation. The householder is 65 or over, the income range is $3,000 to $6,999 (the poverty level is in this range),

and median home value is $38,300.

H = median home value = $38,300

NOI = $3,415 per year[4]

R = NOI/H = $3,415/$38,300 = .0892

By equation (2)

homeowner IRR = R + inflation rate =

.0892 + inflation rate

The values for homeowner IRR are shown in column I of Table 3-1.

In order to find the RM IRRs, RM annuities are calculated based on current mortgage interest rates (10% on May 23, 1986). Annual payments made at the beginning of each year are $995.69 for a male, $687.01 for a female, and $486.33 for a couple. These payments result in a loan balance which grows to $30,640 (80% of $38,300) at life expectancy. The RM IRRs which satisfy equation (3) are shown in column II of Table 3-1. Note that for males and females the RM IRRs are higher than the homeowner IRRs with inflation 4% and above. This is a positive leverage effect, where the growth in NOI and home value outweigh the interest charges on the loan. Positive leverage for couples begins at an inflation rate of 2%.

Transaction costs are now added to test the effect on the relationships. The reversion term in equation (1) becomes $PVFR \times F \times H \times (1+X)^N$ where F is the complement of the sales commission. (For example, F = .94 for a 6% commission.) The homeowner is locked into the investment and can only

Table 3-1

Reverse Mortgage to Life Expectancy

Male: Life Expectancy 14 years

Inflation	I No Sales Homeowner IRR	II Comm. RM IRR	III 6% Sales Homeowner IRR	IV Comm. RM IRR	Annual RM	Total RM
0	8.92%	8.28%	9.49%	8.97%		
2%	10.92	10.83	11.49	11.53	$995.69	$13,940
4%	12.92	13.20	13.49	13.88		
6%	14.92	15.44	15.49	16.12		

Female: Life Expectancy 17 years

Inflation

0%	8.92%	8.43%	9.49%	9.12%		
2%	10.92	10.90	11.49	11.57	$687.01	$11,679
4%	12.92	13.18	13.49	13.85		
6%	14.92	15.36	15.49	16.02		

Couple: Life Expectancy 20 years

Inflation

0%	8.92%	8.54%	9.49%	9.21%		
2%	10.92	10.93	11.49	11.58	$486.33	$9,727
4%	12.92	13.15	13.49	.1379		
6%	14.92	15.28	15.49	.1592		

reinvest the sales proceeds net of the sales commission. Therefore, the last term in equation (1), the amount of the investment, becomes F x H. In Chapter II, it is also shown that the IRR which satisfies the NPV equation with transaction costs is

$$\text{homeowner IRR} = \frac{R}{F} + \text{inflation rate} \qquad (4)$$

For the Orlando SMSA application, this becomes (with a 6% sales commission)

$$\text{homeowner IRR} = \frac{.0892}{.94} + \text{inflation rate}$$

$$= .0949 + \text{inflation rate}$$

This is shown in column III of Table 3-1.

Performing the same modifications as above to equation (3) yields the RM IRRs in column IV of Table 3-1. Note that there is positive leverage for all cases where the inflation rate is 2% or more. This is an encouraging result, which gives reason for further investigation.

Application

Reverse Mortgage with No Transaction Costs

The introductory example in the previous section assumes that all individuals live to life expectancy. The case of the household with a reverse mortgage must be altered for those surviving past life expectancy. The house must be sold to pay the loan balance; subsequent imputed rent and income from the reverse mortgage are lost. The only income stems from the remaining equity invested at the risk-free rate. Columns I through III in Table 3-2 show the cash flow for a

Table 3-2

Cashflow, Reversion, and IRRs
for a Male at 2% Inflation

Age	I Home Cashflow	II RM Benefit	III Total Cashflow[a]	IV Sale Price	V Loan Balance	VI Reversion[b]	VII RM IRR
65	3415	995.7	4411	39066	1095	37971	.1066
66	3483	995.7	4479	39847	2300	37547	.1067
67	3553	995.7	4549	40644	3625	37019	.1068
68	3624	995.7	4620	41457	5083	36374	.1069
69	3697	995.7	4692	42286	6687	35600	.1070
70	3770	995.7	4766	43132	8451	34681	.1072
71	3846	995.7	4842	43995	10391	33604	.1073
72	3923	995.7	4918	44875	12525	32349	.1074
73	4001	995.7	4997	45772	14873	30899	.1076
74	4081	995.7	5077	46687	17456	29232	.1077
75	4163	995.7	5159	47621	20296	27325	.1079
76	4246	995.7	5242	48574	23421	25152	.1080
77	4331	995.7	5327	49545	26859	22686	.1082
78	4418	995.7	5413	50536	30640	19896	.1083
79			1492[c]			19896[d]	.1078
80			1492			19896	.1073
81			1492			19896	.1068
82			1492			19896	.1064
83			1492			19896	.1061
84			1492			19896	.1058
85			1492			19896	.1055
86			1492			19896	.1052
87			1492			19896	.1050
88			1492			19896	.1048
89			1492			19896	.1046
90			1492			19896	.1044
91			1492			19896	.1043
92			1492			19896	.1042
93			1492			19896	.1040
94			1492			19896	.1039
95			1492			19896	.1038
96			1492			19896	.1038
97			1492			19896	.1037
98			1492			19896	.1036
99			1492			19896	.1035

[a]Column I + Column II until end of life expectancy.
[b]Column IV - Column V until end of life expectancy.
[c]7.5% of Column VI.
[d]Reversion reinvested in a bond yielding 7.5%.

male when the inflation rate is 2%. Columns IV through VI track the sale price (growing at the inflation rate), loan balance, and reversion. Reversion after life expectancy is the principal amount of the bond which yields the risk-free rate.

Column VII of Table 3-2 shows the IRR for thirty-five possible holding periods--death at each age from 65 through 99. In order to calculate an expected return for the 35 IRRs, probability of death at each age is required. Mortality rates from the 1980 Commissioners Standard Ordinary Tables (1980 CSO) are used to calculate probabilities of death for each age.[5] Death of a couple is marked by death of the surviving spouse. Summing the products of IRR and death probability gives an expected return for each inflation rate. The expected return for the male in Table 3-2 is 10.67%. Table 3-3 shows the expected returns for males, females, and couples at various inflation rates. These expected returns are the sums of the products of the IRRs for each holding period times the probability of death in the last year of the period. (The calculations in Table 3-2 are one example of the calculations required to arrive at the results shown in Table 3-3.)

Referring to Table 3-3, a household's preference to remain in the home and have a reverse mortgage, compared with other opportunities for investment, depends greatly on the household's expectations for inflation. As an example, the household might have the following expectations.

Table 3-3

Expected Returns for Households with Reverse Mortgage
No Transaction Costs

	RM IRR
Inflation	Male
0	.0836
2%	.1067
4%	.1282
6%	.1487
Inflation	Female
0	.0848
2%	.1076
4%	.1289
6%	.1492
Inflation	Couple
0	.0857
2%	.1084
4%	.1297
6%	.1501

2% inflation 10% probability

4% inflation 70% probability

6% inflation 20% probability

Based on this probability distribution, expected returns and standard deviation can be computed. Standard deviation is a measure of risk, another important part of an investment decision. The following results are for the probability distribution given, which will be referred to as distribution A.

	expected return E(R)	standard deviation S(R)
Male	.1302	.0112
Female	.1308	.0111
Couple	.1317	.0112

These values can now be plotted on risk-return space, with expected return, or E(R), on the vertical axis, and standard deviation, or S(R), on the horizontal axis. The three points are on Figure 3-1 under the letter A.

Other investment alternatives are on the efficient frontier of investments, a line defined by two points. One point is the return on a risk-free asset, which by definition, has a standard deviation of zero. The return on a Treasury Bond maturing in 30 years is the proxy used here for the risk-free asset. The current yield is 7.5% (May 23, 1986), and is shown as point R on Figure 3-1.

The second point defining the efficient frontier of investments is the risk and return on a portfolio of risky securities. The proxy used here is based on the index and dividends for Standard and Poor's 500 stocks. Table 2-1

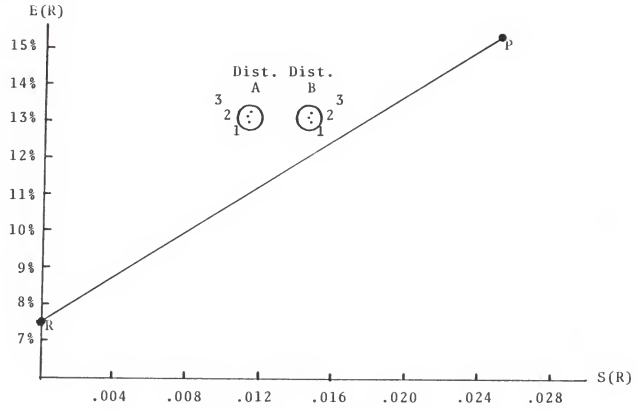

Figure 3-1

Efficient Frontier and Expected Returns--No Transaction Costs

Legend: 1 = Male
2 = Female
3 = Couple

shows the figures used in arriving at E(R) and S(R) for the risky portfolio (shown as point P on Figure 3-1). An investment with a point lying below the efficient frontier should be sold and reinvested at some point on the efficient frontier. The points based on distribution A indicate that the elderly households are receiving an adequate return on home equity.

A second probability distribution, distribution B, is also illustrated. Distribution B also favors the inflation rate continuing at 4%, with much less certainty in the mean.

Distribution B
2% inflation 20% probability
4% inflation 50% probability
6% inflation 30% probability

	expected return E(R)	standard deviation S(R)
Male	.1301	.0147
Female	.1307	.0145
Couple	.1316	.0146

The three points are on Figure 3-1 under the letter B. They also indicate that the households are receiving an adequate return on equity. The next section will test the effect of transaction costs on the relationships.

Reverse Mortgage with Transaction Costs

The transaction cost on the home is the same 6% sales commission used earlier. The returns are shown in Table 3-4. Also as before, the IRRs are based on an investment of $36,002, the equity remaining after the sales commission is paid. The returns on the risky portfolio are lower due to fees on both the buy and sell transactions (see Table 2-2).

Table 3-4

Expected Returns for Households with a Reverse Mortgage
Transaction Costs Included

	RM IRR
Inflation	Male
0	.0902
2%	.1132
4%	.1347
6%	.1553
Inflation	Female
0	.0914
2%	.1141
4%	.1353
6%	.1557
Inflation	Couple
0	.0922
2%	.1148
4%	.1360
6%	.1564

Figure 3-2 shows the results of distributions A and B with the returns from Table 3-4. The relationships have not changed by the introduction of transaction costs--figures 3-1 and 3-2 both indicate that the elderly households receive an adequate return on home equity when participating in a reverse mortgage.

Premium for Life Tenure

Both lenders and households face certain risks by participating in reverse mortgages. Lenders are concerned with the default risk posed by households surviving past life expectancy. Table 3-5 illustrates an example of the potential risk to the lender. The example is based on a male household and 2% inflation. The RM benefit in column I is the same as that shown in Table 3-2. The balance in column II is based on the assumption that all households surviving past life expectancy default. Benefits are no longer being paid; the balance continues to grow after age 78 because of interest charges. Column III shows the net sale price after a 6% commission. The reversion in column IV is the loan balance or net sale price, whichever is less. The IRR in column V is calculated with the lender's investment from column I and the reversion from column IV. The returns to the lender remain at 10% for five years beyond life expectancy. Returns are lower beginning with death at age 84 because the balance is greater than the net sale price. The lender's expected return is less than 10% because of the default risk.

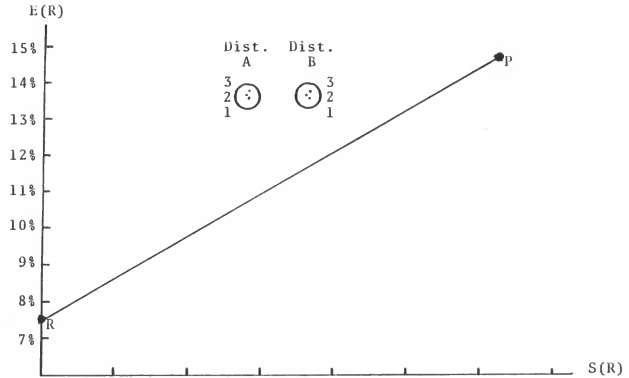

Figure 3-2

Efficient Frontier and Expected Returns With Transaction Costs

Legend: 1 = Male
2 = Female
3 = Couple

Table 3-5

Lender Return--RM Without Premium

Age at Death	I RM Benefit	II Balance	III Net Sale Price	IV Reversion	V Return (IRR)
65	995.7	1095	36722	1095	.1000
66	995.7	2300	37456	2300	.1000
67	995.7	3625	38206	3625	.1000
68	995.7	5083	38970	5083	.1000
69	995.7	6687	39749	6687	.1000
70	995.7	8451	40544	8451	.1000
71	995.7	10391	41355	10391	.1000
72	995.7	12525	42182	12525	.1000
73	995.7	14873	43026	14873	.1000
74	995.7	17456	43886	17456	.1000
75	995.7	20296	44764	20296	.1000
76	995.7	23421	45659	23421	.1000
77	995.7	26859	46572	26859	.1000
78	995.7	30640	47504	30640	.1000
79	0	33704	48454	33704	.1000
80	0	37074	49423	37074	.1000
81	0	40782	50411	40782	.1000
82	0	44860	51420	44860	.1000
83	0	49346	52448	49346	.1000
84	0	54280	53497	53497	.0989
85	0	59708	54567	54567	.0938
86	0	65679	55658	55658	.0893
87	0	72247	56772	56772	.0853
88	0	79472	57907	57907	.0817
89	0	87419	59065	59065	.0785
90	0	96161	60246	60246	.0756
91	0	105777	61451	61451	.0730
92	0	116355	62680	62680	.0706
93	0	127990	63934	63934	.0684
94	0	140789	65213	65213	.0663
95	0	154868	66517	66517	.0645
96	0	170355	67847	67847	.0628
97	0	187391	69204	69204	.0612
98	0	206130	70588	70588	.0597
99	0	226743	72000	72000	.0583

Households are concerned by the risk of being forced to sell their homes in order to pay off the loans. Life tenure can be guaranteed if the household pays a premium to the lender which will result in an expected return to the lender of 10%. The example shown in Table 3-5 is continued in Table 3-6. Columns II through IV remain the same. The RM benefit in column I is the $995.69 payment less a $37 premium for life tenure. The debt pattern remains unchanged. Column V shows returns that are higher than 10% in the early years due to a lower net investment to the lender. The expected return for the IRRs in column V is 10%. (The probabilities of death must be weighted for the amount of the investment.[6]) The premium for a female household is $24 and the premium for a couple household is only $8.

Premium for Lifetime Annuity

Many households would like the option to accept a lower lifetime annuity rather than an annuity that stops at life expectancy. Table 3-7 continues the example of a male household and 2% inflation. The RM benefit in column I is the $995.69 payment less a premium of $81.50. The balance in column II is greater after life expectancy (compared to Table 3-6) because payments continue; premiums to the lender continue as well. Returns in column V are greater in the early years due to the lower net investment to the lender. The expected return for the IRRs in column V is 10%. Equivalent premiums for female and couple households are $45 and $14.50 respectively.

Table 3-6

Lender Return--Premium for Life Tenure

Age at Death	I RM Benefit	II Balance	III Net Sale Price	IV Reversion	V Return (IRR)
65	958.7	1095	36722	1095	.1425
66	958.7	2300	37456	2300	.1276
67	958.7	3625	38206	3625	.1203
68	958.7	5083	38970	5083	.1160
69	958.7	6687	39749	6687	.1131
70	958.7	8451	40544	8451	.1110
71	958.7	10391	41355	10391	.1095
72	958.7	12525	42182	12525	.1083
73	958.7	14873	43026	14873	.1074
74	958.7	17456	43886	17456	.1066
75	958.7	20296	44764	20296	.1060
76	958.7	23421	45659	23421	.1055
77	958.7	26859	46572	26859	.1050
78	958.7	30640	47504	30640	.1046
79	0	33704	48454	33704	.1042
80	0	37074	49423	37074	.1038
81	0	40782	50411	40782	.1035
82	0	44860	51420	44860	.1032
83	0	49346	52448	49346	.1030
84	0	54280	53497	53497	.1017
85	0	59708	54567	54567	.0964
86	0	65679	55658	55658	.0917
87	0	72247	56772	56772	.0876
88	0	79472	57907	57907	.0839
89	0	87419	59065	59065	.0806
90	0	96161	60246	60246	.0776
91	0	105777	61541	61451	.0748
92	0	116355	62680	62680	.0724
93	0	127990	63934	63934	.0701
94	0	140789	65213	65213	.0680
95	0	154868	66517	66517	.0661
96	0	170355	67847	67847	.0643
97	0	187391	69204	69204	.0627
98	0	206130	70588	70588	.0611
99	0	226743	72000	72000	.0597

Table 3-7

Reverse Mortgage with Lifetime Annuity

Age at Death	I RM Benefit	II Balance	III Net Sale Price	IV Reversion	V Return (IRR)
65	914.2	1095	36722	1095	.1981
66	914.2	2300	37456	2300	.1631
67	914.2	3625	38206	3625	.1462
68	914.2	5083	38970	5083	.1362
69	914.2	6687	39749	6687	.1296
70	914.2	8451	40544	8451	.1249
71	914.2	10391	41355	10391	.1215
72	914.2	12525	42182	12525	.1188
73	914.2	14873	43026	14873	.1167
74	914.2	17456	43886	17456	.1149
75	914.2	20296	44764	20296	.1135
76	914.2	23421	45659	23421	.1123
77	914.2	26859	46572	26859	.1113
78	914.2	30640	47504	30640	.1104
79	914.2	34799	48454	34799	.1096
80	914.2	39374	49423	39374	.1090
81	914.2	44407	50411	44407	.1084
82	914.2	49943	51420	49943	.1078
83	914.2	56032	52448	52448	.1017
84	914.2	62731	53497	53497	.0940
85	914.2	70099	54567	54567	.0872
86	914.2	78205	55658	55658	.0812
87	914.2	87120	56772	56772	.0759
88	914.2	96927	57907	57907	.0712
89	914.2	107716	59065	59065	.0670
90	914.2	119582	60246	60246	.0632
91	914.2	132636	61451	61451	.0597
92	914.2	146995	62680	62680	.0566
93	914.2	162789	63934	63934	.0538
94	914.2	180164	65213	65213	.0512
95	914.2	199275	66517	66517	.0489
96	914.2	220298	67847	67847	.0467
97	914.2	243423	69204	69204	.0448
98	914.2	268861	70588	70588	.0430
99	914.2	296842	72000	72000	.0413

Table 3-8 shows the IRRs for a reverse mortgage with life tenure compared to the basic reverse mortgage with no premiums deducted. In all cases the returns with life tenure are greater. Another result of life tenure is that positive leverage is achieved at lower inflation rates. Therefore, in the Orlando example, households can pay a premium for life tenure with two positive results: the risk of losing the home is removed; and the return on home equity is increased. The reverse mortgage should satisfy two requirements: the returns should be equal to or greater than the returns on the efficient frontier; and the returns should be equal to or greater than the homeowner IRRs. All of the expected returns in Table 3-8 are greater than the returns on the efficient frontier.

Conclusion

This paper develops a model which can compare return on home equity with no reverse mortgage, return on home equity with a standard reverse mortgage, and return on home equity with life tenure guaranteed. The application for the Orlando SMSA shows a situation in which the reverse mortgage with life tenure is better than the standard reverse mortgage; life tenure has benefits for lenders and households alike. Lenders receive a premium which eliminates default risk for expected rates of inflation and mortality. Households are relieved of the risk of losing the home and receive a higher return on home equity than is possible with a standard reverse mortgage.

Table 3-8

Homeowner IRRs and Reverse Mortgage IRRs

	Homeowner IRR	RM IRR	RM IRR With Life Tenure
Inflation	Male		
0	.0949	.0902	.0972
2%	.1149	.1132	.1187
4%	.1349	.1347	.1394
6%	.1549	.1553	.1597
E(R)*	.1369	.1367	.1414
Inflation	Female		
0	.0949	.0914	.0968
2%	.1149	.1141	.1182
4%	.1349	.1353	.1387
6%	.1549	.1557	.1588
E(R)*	.1369	.1380	.1395
Inflation	Couple		
0	.0949	.0922	.0956
2%	.1149	.1148	.1173
4%	.1349	.1360	.1380
6%	.1549	.1564	.1582
E(R)*	.1369	.1380	.1400

*Based on distribution A.

Households should make two demands of a reverse mortgage. First, the point representing the risk-return relationship of the reverse mortgage should lie on or above the efficient frontier. Second, the return with the reverse mortgage should be equal to or greater than the return on the home without a reverse mortgage.

Notes

1. Other versions of the instrument include variable interest rates, graduated payments, payments indexed to inflation, fixed terms of shorter duration, and lifetime annuities.

2. The details of the derivation of NOI are in Chapter II.

3. This results in an exponential-curve pattern of change, where X is the ratio of the increase in income for any period to the income for the previous period.

4. The details for arriving at the value of NOI are in Chapter II.

5. New versions of the tables appear in Transactions of the Society of Actuaries as they become available.

6. This is required because the lender's investment does not involve a lump sum at the beginning of the contract, but a series of payments. The factor used to adjust for this feature is the quotient of the present value of payments received divided by the present value of the maximum number of payments. (Then all of the probabilities must be adjusted in order that they sum to one, or 100%.) This linear weighting may introduce a slight inaccuracy compared to the approach using mortality rates.

CHAPTER IV

PREMIUMS AND SUBSIDIES ON REVERSE SHARED-APPRECIATION MORTGAGES

This paper investigates the economics of a federal insurance program for reverse shared-appreciation mortgages (RSAMs). An advantage of a federal program is that premiums are collected from all regions of the country while claims are paid only to those regions suffering losses.

The first section of the paper deals with lender participation. The minimum interest which will yield the lender's required return is found. The second section deals with household participation and the maximum interest rate which the household is willing to pay is found. The difference between the two rates is the maximum insurance premium which can be charged, and the premium is used in the third section to develop a self-subsidizing program. The final section contains the concluding remarks.

The example program in this paper is designed to work at an appreciation rate of 2%. All figures on elderly households are based on the Orlando Standard Metropolitan Statistical Area (SMSA) and incomes in the range of $3,000 to $6,999--the range which includes the poverty level. In this paper benefits from the RSAMs start at age 65 and stop at life expectancy--life tenure is guaranteed.

Previous works look at the economic returns to lenders and to insurance premiums for reverse mortgages (not RSAMs) based on expected losses to lenders [2-6]. The premium discussed above can be found only if the economic returns to the elderly homeowners are quantified as they are in the second section of this paper.

Lender Participation

The requirement for lender participation is that the expected return from the RSAM equals the market return for mortgage loans (10% on May 23, 1986). The return from the RSAM is based on interest + contingent interest (appreciation). Life tenure is guaranteed, resulting in no special requirement to have a premium for default risk for households living beyond life expectancy.

The guarantee of life tenure requires the use of a mortality distribution in order to find the expected return to the lender. One approach is to start with an entrant group, for example, 100,000 males aged 65, and find a single expected return for the group with the use of mortality rates [3]. Mortality rates are based on the number of survivors at each age after entry. The approach used here is more revealing, as a return is found for death at each age. Probabilities of death at each age are then used to find an expected return for the group. Probabilities of death are based on the original size of the group, as opposed to the number of survivors. (The probabilities of death must be weighted for the amount of the investment.[1])

The RSAM benefit, an annual payment to the household at the beginning of the year, is found by[2]

$$\text{RSAM benefit} = \text{loan amount} / \frac{[(1+R) \times ((1+R)^{L} - 1)]}{R} \quad (1)$$

where R = interest rate

L = life expectancy

The denominator is a sinking fund factor for annual payments made at the beginning of the year. The RSAM benefit in column I of Table 4-1 is based on a loan amount of $30,640 (80% of a $38,300 home value at the beginning of the holding period), and a male life expectancy of 14 years. The interest rate is changed in trial and error fashion until the expected return to the lender is at the desired value. In the example, an interest rate of 5.36% plus appreciation (2%) yields an expected return of 10%. Column II tracks the balance based on the interest rate of 5.36%, column III shows the appreciation, and column IV is the total RSAM balance which is the sum of columns II and III. Net sale price in column V is the value based on 2% appreciation less a 6% sales commission. In calculating the lender's internal rate of return (IRR), shown in column VII, the lender's reversion is the RSAM balance or net sale price, whichever is less.

The trends in column VII reveal the true nature of the RSAM. The IRRs to the lender start at a high of 58.29% for death at age 65 and fall to a low of 4.49% for death at age 99. The expected return of 10% is found by summing the

Table 4-1

Lender Return--Male Household

Age at Death	I RSAM Benefit	II Balance	III Appreciation	IV Total RSAM Balance	V Net Sale Price	VI Reversion	VII Return (IRR)
65	1447	1525	766	2291	36722	2291	.5829
66	1447	3131	1547	4678	37456	4678	.3662
67	1447	4824	2344	7168	38206	7168	.2726
68	1447	6607	3157	9764	38970	9764	.2205
69	1447	8486	3986	12472	39749	12472	.1873
70	1447	10465	4832	15297	40544	15297	.1644
71	1447	12551	5695	18246	41355	18246	.1476
72	1447	14749	6575	21323	42182	21323	.1348
73	1447	17064	7472	24536	43026	24536	.1247
74	1447	19503	8387	27891	43886	27891	.1166
75	1447	22073	9321	31394	44764	31394	.1099
76	1447	24781	10274	35055	45659	35055	.1042
77	1447	27634	11245	38879	46572	38879	.0995
78	1447	30640	12236	42876	47504	42876	.0954
79	0	32282	13247	45529	48454	45529	.0919
80	0	34013	14278	48290	49423	48290	.0890
81	0	35836	15329	51165	50411	51165	.0851
82	0	37757	16402	54158	51420	51420	.0799
83	0	39780	17496	57276	52448	52448	.0754
84	0	41912	18612	60524	53497	53497	.0715
85	0	44159	19750	63909	54567	54567	.0681
86	0	46526	20911	67437	55658	55658	.0652
87	0	49020	22095	71115	56772	56772	.0625

88	0	51647	23303	74950	57907	57907	.0602
89	0	54415	24535	78951	59065	59065	.0581
90	0	57332	25792	83124	60246	60246	.0562
91	0	60405	27074	87479	61451	61451	.0544
92	0	63643	28381	92024	62680	62680	.0529
93	0	67054	29715	96769	63934	63934	.0514
94	0	70648	31075	101723	65213	65213	.0501
95	0	74435	32463	106898	66517	66517	.0489
96	0	78425	33878	112303	67847	67847	.0478
97	0	82628	35321	117950	69204	69204	.0468
98	0	87057	36794	123851	70588	70588	.0458
99	0	91723	38296	130019	72000	72000	.0449

products of the IRRs in column VII with the probabilities of death. Calculations of losses are not necessary--reversion values which are less than the RSAM balances are reflected in lower returns. Note that the return at life expectancy (age 78) is not equal to the expected return for the group. In other words, if the interest rate is calculated based on life expectancy, the expected returns will be higher and biased in the lender's favor.

Tables 4-2 and 4-3 show the same figures for females and couples. The interest rates required to give the lender an expected return of 10% are 5.59% and 5.92% for females and couples respectively.

Household Participation

The purpose of this section is to find the maximum interest rate that the household is willing to pay. The homeowner should not participate in the RSAM if the internal rate of return to the homeowner without the RSAM (homeowner IRR) is greater than the internal rate of return to the homeowner with the RSAM (RSAM IRR). (It is also necessary that the household receive an adequate return on home equity relative to investment alternatives. This requirement is satisfied if the point representing the risk-return relationship lies on or above the efficient frontier of investments. See Chapter II.) Therefore, the maximum interest rate is found by finding the rate which equates homeowner IRR and RSAM IRR.

Table 4-2

Lender Return--Female Household

Age At Death	I RSAM Benefit	II Balance	III Appreciation	IV Total RSAM Balance	V Net Sale Price	VI Reversion	VII Return (IRR)
65	1066	1126	766	1892	36722	1892	.7742
66	1066	2315	1547	3862	37456	3862	.4667
67	1066	3570	2344	5915	38206	5915	.3405
68	1066	4896	3157	8053	38970	8053	.2711
69	1066	6296	3986	10282	39749	10282	.2275
70	1066	7774	4832	12606	40544	12606	.1976
71	1066	9334	5695	15029	41355	15029	.1758
72	1066	10982	6575	17557	42182	17557	.1593
73	1066	12722	7472	20194	43026	20194	.1463
74	1066	14559	8387	22947	43886	22947	.1358
75	1066	16499	9321	25820	44764	25820	.1272
76	1066	18547	10274	28821	45659	28821	.1200
77	1066	20710	11245	31955	46572	31955	.1139
78	1066	22994	12236	35230	47504	35230	.1086
79	1066	25405	13247	38652	48454	38652	.1041
80	1066	27951	14278	42229	49423	42229	.1001
81	1066	30640	15329	45969	50411	45969	.0966
82	0	32353	16402	48755	51420	48755	.0936
83	0	34161	17496	51657	52448	51657	.0910
84	0	36071	18612	54683	53497	53497	.0869
85	0	38087	19750	57837	54567	54567	.0823
86	0	40216	20911	61127	55658	55658	.0782
87	0	42464	22095	64560	56772	56772	.0747

88	0	44838	23303	68141	57907	57907	.0715
89	0	47345	24535	71880	59065	59065	.0887
90	0	49991	25792	75783	60246	60246	.0661
91	0	52786	27074	79859	62451	61451	.0638
92	0	55736	28381	84118	62680	62680	.0617
93	0	58852	29715	88567	63934	63934	.0598
94	0	62142	31075	93217	65213	65213	.0581
95	0	65616	32463	96078	66517	66517	10565
96	0	69284	33878	103162	67847	67847	.0550
97	0	73157	35321	108478	69204	69204	.0537
98	0	77246	36794	114040	70588	70588	.0524
99	0	81564	38296	119860	72000	72000	.0512

Table 4-3

Lender Return--Couple Household

Age At Death	I RSAM Benefit	II Balance	III Appreciation	IV Total RSAM Balance	V Net Sale Price	VI Reversion	VII Return (IRR)
65	793.2	840.1	766	1606	36722	1606	1.025
66	793.2	1730	1547	3277	37456	3277	.5933
67	793.2	2673	2344	5017	38206	5017	.4226
68	793.2	3671	3157	6828	38970	6828	.3315
69	793.2	4728	3986	8715	39749	8715	.2751
70	793.2	5848	4832	10680	40544	10680	.2367
71	793.2	7035	5695	12729	41355	12729	.2089
72	793.2	8291	6574	14866	42182	14866	.1879
73	793.2	9622	7472	17094	43026	17094	.1714
74	793.2	11032	8387	19419	43886	19419	.1582
75	793.2	12525	9321	21846	44764	21846	.1474
76	793.2	14107	10274	24380	45659	24380	.1384
77	793.2	15782	11245	27027	46572	27027	.1307
78	793.2	17566	12236	29793	47504	29793	.1242
79	793.2	19436	13247	32683	48454	32683	.1185
80	793.2	21427	14378	35704	49423	35704	.1186
81	793.2	23535	15329	38865	50411	38865	.1092
82	793.2	25769	16402	42171	51420	42171	.1054
83	793.2	26134	17496	45630	52448	45630	.1019
84	793.2	30640	18612	49252	53497	49252	.1988
85	0	32454	19750	52204	54567	52204	.0961
86	0	34375	20911	55286	55658	55286	.1036
87	0	36410	22095	58505	56772	56772	.0894

88	0	38566	23303	61869	57907	57907	.0852
89	0	40849	24535	65384	59065	59065	.0815
90	0	43267	25792	69059	60246	60246	.0782
91	0	45828	27074	72902	61451	61451	.0752
92	0	48541	28381	76923	62680	62680	.0724
93	0	51415	29715	81130	63934	63934	.0700
94	0	54459	31075	85534	65213	65213	.0677
95	0	57653	32463	90145	66517	66517	.0657
96	0	61098	33878	94976	67847	67847	.0638
97	0	64715	35321	100036	69204	69204	.0620
98	0	68546	36794	105340	70588	70588	.0604
99	0	72604	38296	110899	72000	72000	.0589

Homeowner IRR is found by (see Chapter II)

$$\text{homeowner IRR} = \frac{R}{F} + \text{appreciation} \qquad (2)$$

where R = overall capitalization rate

= net operating income (NOI)/home value

F = 1 - sales commission

The investment amount used in calculating the IRR reflects the actual equity available to one who is locked into the investment--home value less the sales commission. Equation (2) holds for any life expectancy.

Numerical values derived from the most recent Annual Housing Survey for the Orlando SMSA are used as an application of equation (2). The householder is 65 or over, the income range is $3,000 to $6,999 (the poverty level is in this range), and median home value is $38,300.

H = median home value = $38,300

NOI = $3,415 per year

R = NOI/H = $3,415/$38,300 = .0892

By equation (2), with a 6% sales commission and 2% appreciation

$$\text{homeowner IRR} = \frac{.0892}{1-.06} + .02 = .1149$$

The interest rate in equation (1) is changed in trial and error fashion until RSAM IRR = homeowner IRR = .1149. Table 4-4 shows the cash flows needed to find the RSAM IRR for death at each age. Home income in column I is the NOI increasing at the rate of appreciation. (The household has no mortgage and there is no tax on a homeowner's imputed

Table 4-4

Return for Male Household

Age At Death	I Home Income	II RSAM Benefit	III Total Income	IV Balance	V Appreciation	VI Total RSAM Balance	VII Net Sale Price	VIII Reversion	IX Return (IRR)
65	3415	1354	4769	1438	766	2204	36722	34518	.0912
66	3483	1354	4837	2964	1547	4512	37456	32945	.0928
67	3553	1354	4907	4586	2344	6930	38206	31275	.0944
68	3624	1354	4978	6308	3157	9465	38970	29505	.0960
69	3697	1354	5050	8137	3986	12123	39749	27626	.0977
70	3770	1354	5124	10079	4832	14911	40544	25633	.0994
71	3848	1354	5200	12141	5695	17836	41355	23519	.1012
72	3923	1354	5276	14332	6575	20906	42182	21276	.1030
73	4001	1354	5355	16658	7472	24130	43026	18896	.1048
74	4081	1354	5435	19129	8387	27516	43886	16370	.1066
75	4163	1354	5517	21752	9321	31073	44764	13691	.1085
76	4246	1354	5600	24538	10274	34812	45659	10847	.1104
77	4331	1354	5685	27497	11245	38743	46572	7830	.1123
78	4418	1354	5771	30640	12236	42876	47504	4628	.1141
79	4506	0	4506	32540	13247	45786	48454	2668	.1161
80	4596	0	4596	34557	14278	48835	49423	588.2	.1179
81	4688	0	4688	36700	15329	52029	50411	0	.1207
82	4782	0	4782	38975	16402	55377	51420	0	.1236
83	4877	0	4877	41392	17496	58887	52448	0	.1260
84	4975	0	4975	43958	18612	62570	53497	0	.1280
85	5075	0	5075	46683	19750	66433	54567	0	.1298
86	5176	0	5176	49578	20911	70489	55658	0	.1314
87	5280	0	5280	52651	22095	74747	56772	0	.1327

88	5385	0	5385	55916	23303	79219	57907	0	.1338
89	5493	0	5493	59382	24535	83918	59065	0	.1348
90	5603	0	5603	63064	25792	88856	60246	0	.1357
91	5715	0	5715	66974	27074	94048	61451	0	.1365
92	5829	0	5829	71127	28381	99508	62680	0	.1371
93	5946	0	5946	75536	29715	105251	63934	0	.1377
94	6065	0	6065	80220	31075	111295	65213	0	.1382
95	6186	0	6186	85193	32463	117656	66517	0	.1387
96	6310	0	6310	90475	33878	124353	67847	0	.1391
97	6436	0	6436	96085	35321	131406	69204	0	.1394
98	6564	0	6564	102042	36794	138836	70588	0	.1397
99	6696	0	6696	108369	38296	146664	72000	0	.1400

rent.) Column II is the RSAM benefit from equation (1); column III is the sum of columns I and II.

Columns IV through VII show the calculations necessary to arrive at reversion, column VIII. The interest rate which equates homeowner IRR and RSAM IRR for a male with a life expectancy of 14 years is 6.2%. The balance in column IV results from the RSAM payments and interest compounding at 6.2%. Column V tracks the appreciation; the RSAM balance in column VI is the sum of columns IV and V. The net sale price, column VII, is the value of the home growing at the appreciation rate of 2% less a 6% sales commission. Reversion, column VIII, is net sale price less the RSAM balance--when the RSAM balance exceeds the net sale price, reversion becomes zero. (This is consistent with the method previously used in calculating reversion for the lender.)

The RSAM IRR in column IX is the internal rate of return to the household with the RSAM utilizing the annual cash flows from column III and reversion from column VIII. The expected return from the IRRs in column IX, using probabilities of death at each age for males, is 11.49%.

The previous section shows that the minimum interest rate which the lender can charge a male and have an expected return of 10% is 5.36%. The maximum interest rate which a male household will pay is 6.2%. The difference, .84% is the maximum premium which a male is willing to pay.

Table 4-5 shows the cash flows for a female with a life expectancy of 17 years. The interest rate equating homeowner

Table 4-5

Return for Female Household

Age At Death	I Home Income	II RSAM Benefit	III Total Income	IV Balance	V Appreciation	VI Total RSAM Balance	VII Net Sale Price	VIII Reversion	IX Return (IRR)
65	3415	966.9	4382	1031	766	1797	36722	34925	.0918
66	3483	966.9	4450	2129	1547	3676	37456	33780	.0931
67	3553	966.9	4520	3800	2344	5644	38206	32561	.0945
68	3624	966.9	4591	4548	3157	7705	33970	31265	.0958
69	3697	966.9	4663	5878	3986	9865	39749	29884	.0972
70	3770	966.9	4737	7296	4832	12128	40544	28416	.0986
71	3846	966.9	4813	8808	5695	14502	41355	26853	.1001
72	3923	966.9	4890	10419	6575	16993	42182	25189	.1015
73	4001	966.9	4968	12136	7472	19608	43026	23418	.1029
74	4081	966.9	5048	13966	8387	22354	43886	21532	.1044
75	4163	966.9	5130	15917	9321	25239	44764	19525	.1058
76	4246	966.9	5213	17997	10274	28271	45659	17389	.1073
77	4331	966.9	5298	20213	11245	31459	46572	15114	.1087
78	4418	966.9	5385	22576	12236	34812	47504	12692	.1101
79	4506	966.9	5473	25094	13247	38341	48454	10113	.1115
80	4596	966.9	5563	27779	14278	42056	49423	7367	.1129
81	4688	966.9	5655	30640	15329	45969	50411	4442	.1142
82	4782	0	4782	32659	16402	49061	51420	2359	.1156
83	4877	0	4877	34811	17496	52307	52448	140.8	.1168
84	4975	0	4975	37105	18612	55717	53497	0	.1190
85	5075	0	5075	39551	19750	59301	54567	0	.1210
86	5176	0	5176	42157	20911	63068	55658	0	.1227
87	5280	0	5280	44935	22095	67031	56772	0	.1242

88	5385	0	5385	47897	23303	71200	57907	0	.1255
89	5493	0	5493	51053	24535	75588	59065	0	.1266
90	5603	0	5603	54417	25792	80209	60246	0	.1276
91	5715	0	5715	56003	27074	85077	61451	0	.1284
92	5829	0	5829	61826	28381	90207	62680	0	.1292
93	5946	0	5946	65900	29715	95615	63934	0	.1298
94	6065	0	6065	70243	31075	101318	65213	0	.1304
95	6186	0	6186	74872	32463	107335	66517	0	.1309
96	6310	0	6310	79806	33878	113684	67847	0	.1314
97	6436	0	6436	85065	35321	120387	69204	0	.1318
98	6564	0	6564	90671	36794	127465	70588	0	.1322
99	6696	0	6696	96646	38296	134942	72000	0	.1325

IRR to RSAM IRR is 6.59%. The minimum interest rate which a lender can charge, found in the previous section, is 5.59% for an expected return of 10%. Therefore, the maximum premium which a female is willing to pay is 1%.

Table 4-6 shows the cash flows for a couple with a life expectancy of 20 years. Life expectancy for a couple is based on death of the surviving spouse. The maximum interest rate for couples is 6.82%. The maximum premium for couples, subtracting the interest rate of 5.92% from the previous section, is .9%.

A Self-Subsidizing RSAM Program

This section investigates the feasibility of a self-subsidizing RSAM program. Without the self-subsidizing feature, a federally insured program would have an expected loss of zero simply by adjusting interest rates to give lenders an expected return equal to the required return. The objective here is to maximize the impact on poverty. The method employed is to charge premiums on households with high-value homes in order to subsidize households with low-value homes. The maximum premium which can be charged is found as demonstrated in the previous section.

The impact on poverty is measured here by the ability of the program to have a RSAM benefit equal to the mean deficit below poverty. For the group under consideration, the mean deficit is $1,134 per year for males, $1,158 per year for females, and $1,674 per year for couples. Note from the outset that the task will be particularly difficult for couples, as the RSAM benefits are lower for couples than for individuals.

Table 4-6

Return for Couple Household

Age At Death	I Home Income	II RSAM Benefit	III Total Income	IV Balance	V Appreciation	VI Total RSAM Balance	VII Net Sale Price	VIII Reversion	IX Return (IRR)
65	3415	713.6	4129	762.2	766	1528	36722	35194	.0922
66	3483	713.6	4197	1576	1547	3124	37456	34333	.0934
67	3553	713.6	4267	2446	2344	4790	38206	33415	.0946
68	3624	713.6	4338	3375	3157	6532	38970	32437	.0959
69	3697	713.6	4410	4368	3986	8354	39749	31395	.0971
70	3770	713.6	4484	5428	4832	10260	40544	30284	.0983
71	3846	713.6	4559	6560	5695	12255	41355	29100	.0996
72	3923	713.6	4636	7770	6575	14344	42182	27838	.1008
73	4001	713.6	4715	9062	7472	16534	43026	26492	.1020
74	4081	713.6	4795	10442	8387	18829	43886	25057	.1033
75	4165	713.6	4876	11916	9321	21238	44764	23526	.1045
76	4246	713.6	4960	13491	10274	23765	45659	21894	.1057
77	4331	713.6	5045	15174	11245	26419	46572	20154	.1069
78	4418	713.6	5131	16971	12236	29207	47504	18297	.1080
79	4506	713.6	5220	18890	13247	32137	43454	16317	.1092
80	4596	713.6	5310	20941	14278	35218	49423	14205	.1103
81	4688	713.6	5402	23131	15329	38460	50411	11951	.1114
82	4782	713.6	5495	25471	16402	41873	51420	9547	.1125
83	4877	713.6	5591	27970	17496	45466	52448	6982	.1135
84	4975	713.6	5689	30640	18612	49252	53497	4245	.1145
85	5075	0	5075	32730	19750	52480	54567	2087	.1155
86	5176	0	5176	34962	20911	55873	55658	0	.1165
87	5280	0	5280	37346	22095	59441	56772	0	.1181

88	5385	0	5385	39893	23303	63196	57907	0	.1195
89	5493	0	5493	42614	24535	67149	59065	0	.1207
90	5603	0	5603	45520	25792	71312	60246	0	.1218
91	5715	0	5715	48625	27074	75698	61451	0	.1227
92	5829	0	5829	51941	28381	80322	62680	0	.1235
93	5946	0	5946	55483	29715	85198	63934	0	.1242
94	6065	0	6065	59267	31075	90342	65213	0	.1249
95	6186	0	6186	63309	32463	95772	66517	0	.1255
96	6310	0	6310	67627	33878	101505	67847	0	.1260
97	6436	0	6436	72239	35321	107561	69204	0	.1264
98	6564	0	6564	77166	36794	113960	70588	0	.1268
99	6696	0	6696	82428	38296	120724	72000	0	.1272

The problem, restated, is to maximize the number of households receiving RSAM benefits equal to the mean deficit below poverty, with the following constraints: the expected loss of the program must be less than or equal to zero; premiums cannot be charged which are greater than the maximum premiums found in the previous section. The discussion requires references to Table 4-7.

The columns in Table 4-7 are for the home values found in the Annual Housing Survey; the figures shown are the midpoints for each value range. The probability rows show the joint probability that the home has the value for that column and that the household is male, female, or couple. The group under consideration has 14.5% male individuals, 59.7% female individuals, and 25.8% couple households. These probabilities will be used to find the expected value of the gain or loss for the program. The unadjusted payments are based on home value with no premium or subsidy.

The boxes which contain only dashes represent households which cannot be helped by the program. The criterion for this is the inability to have an RSAM benefit equal to the mean deficit below poverty with an interest rate as low as zero. This constitutes approximately 38% of the households.

The adjusted payments are more or less than the unadjusted payments depending upon whether the household is paying a premium or receiving a subsidy. The households receiving a subsidy can be identified by a value in parentheses in the row labeled "present value of premium (subsidy)."

Table 4-7

Self-Subsidizing RSAM Program

Value	$17,500	$22,500	$27,500	$32,500	$37,500	$45,000
Male:						
Probability	.0137	.0153	.0153	.0153	.0137	.0305
Unadjusted payment	--	850	1039	1228	1417	1700
Adjusted payment	--	1143	1142	1149	1325	1591
Present value of premium(subsidy)	--	(1983)	(698)	538	620	744
% premium (subsidy)	--	(3.8)	(1.2)	.84	.84	.84
Female:						
Probability	.0566	.0628	.0628	.0628	.0566	.1257
Unadjusted payment	--	--	--	905	1044	1253
Adjusted payment	--	--	--	1172	1173	1205
Present value of premium(subsidy)	--	--	--	(2066)	(994)	371
% premium (subsidy)	--	--	--	(2.7)	(1.2)	.4
Couple:						
Probability	.0244	.0272	.0272	.0272	.0244	.0543
Unadjusted payment	--	--	--	--	--	--
Adjusted payment	--	--	--	--	--	--
Present value of premium(subsidy)	--	--	--	--	--	--
% premium (subsidy)	--	--	--	--	--	--

$55,000	$67,500	$87,500	$112,500	$137,500	$175,000
.0122	.0153	.0076	.0031	.0015	.0015
2078	2250	3306	4251	5195	6612
1944	2386	3093	3976	4860	6185
910	1117	1447	1861	2275	2895
.84	.84	.84	.84	.84	.84
.0503	.0628	.0314	.0126	.0063	.0063
1531	1879	2436	3132	3828	4873
1388	1704	2209	2840	3471	4418
1106	1357	1759	2262	2764	3518
1.0	1.0	1.0	1.0	1.0	1.0
.0217	.0272	.0136	.0054	.0027	.0027
1139	1398	1812	2330	2848	3624
1695	1701	1729	2096	2562	3260
(4874)	(2653)	727	2049	2504	3187
(3.5)	(1.7)	.4	.9	.9	.9

To arrive at the value of the premiums, for each age the expected value of the premium is found by the product of the premium and the probability of being alive to pay the premium. Summing the present value of the products gives the present value of expected premiums.

The expected gain or loss of the program is obtained by summing the products of present values and probabilities shown in the table. It is a trial and error process. In some cases, households that met the criteria above (ability to receive a benefit equal to the mean deficit below poverty) must be eliminated because the program "can't afford" them (inclusion results in an expected loss for the program). The program shown in Table 4-7 has the following expected gains (losses):

male households	+51.3
female households	+124.6
couple households	-141.5
Overall Program	34.4

Conclusion

This paper develops a model for calculating the maximum premium on the RSAM for a low-income elderly household. The minimum interest which will yield the lender's required return is found by holding appreciation constant and calculating the return for death at each age. Returns are very high with early deaths, decrease rapidly at first, then more gradually. The return for death at each age, together with the probability of death at each age, yields an expected return.

The maximum interest rate which the household is willing to pay is found by finding the interest rate which equates RSAM IRR to homeowner IRR. The difference between the two rates is the maximum premium which can be charged.

The present value of the premiums depends on the characteristics of the participants. The characteristics of importance are the probability of death at each age (based on male, female, or couple household) and home value. An example is presented using the characteristics of the low-income elderly in the Orlando SMSA.

The objective in the Orlando example is to pay a benefit which is no less than the mean deficit below poverty. Households with high home values pay premiums in order to subsidize households with low home values. The objective can be met for 42% of the households without subsidies, and an additional 20% with subsidies. A subsidy from the government is required to meet the objective for the remaining 38%.

Notes

1. This is required because the lender's investment does not involve a lump sum at the beginning of the contract, but a series of payments. The return for a household not surviving to life expectancy can not have the same weight as the return for a household receiving the maximum number of payments. The factor used to adjust for this feature is the quotient of the present value of payments received divided by the present value of the maximum number of payments. (Then all of the probabilities must be adjusted in order that they sum to one, or 100%.) This linear weighting may introduce a slight inaccuracy compared to the approach using mortality rates.

2. See Appendix A for lifetime annuities.

CHAPTER V

SUMMARY

This study addresses the question of the potential of reverse mortgages to impact the poverty problem. A methodology is developed to quantify the economic returns to the low-income elderly. The methodology requires two components. The first is a cash flow model to find the internal rate of return on home equity. The derivation in Chapter II shows that this can be reduced to a simple algebraic equation for an elderly homeowner with no mortgage and no income taxes. The second component examines the return on home equity in a portfolio theory framework to determine whether the return is adequate compared to investment alternatives. Application of the model to the Orlando SMSA shows that homeowners there do receive an adequate return on home equity. The results in other areas could be different, for example, areas where the property tax burden is greater.

Chapter III extends the cash flow model to include a household participating in a standard RAM and a RAM with a premium for life tenure. The internal rate of return on home equity cannot be reduced to an algebraic equation with the reverse mortgage added to the model; the standard iterative procedure is required. The return with the reverse mortgage must equal or exceed both of the standards developed in

Chapter II: the return must equal or exceed the return on home equity for a homeowner without a reverse mortgage. In the Orlando application, the results for the standard RAM are mixed; return on home equity for a male is reduced by a RAM, while returns for female and couple households are increased (based on an example probability distribution for expectations of inflation). The standard RAM involves the risk of a forced sale to pay off the loan balance, a risk that might be considered too great for the objective of converting equity into income. The Orlando example shows that a RAM with a premium for life tenure increases return on home equity for all levels of inflation of zero or above (compared to a standard RAM). This adds economic support for the intuitive fears about fixed-term RAMs (for the Orlando example).

Chapter IV extends the model to include a household participating in the RSAM, which features life tenure. The two standards developed in Chapter II are applied again as in Chapter III. A self-subsidizing RSAM program is investigated. The objective in the Orlando example is to pay a benefit which is no less than the mean deficit below poverty. The RSAM can meet the objective for 42% of the households. The objective cannot be met for the other 58% because home values are too low. However, households with high-value homes can pay a premium and still receive an adequate return on home equity. When this is done, the objective can be met for an additional 20% of the households with subsidies

covered by the premiums. The objective for the remaining 38% can only be met with a subsidy from the government.

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APPENDIX A

PREMIUM FOR LIFETIME ANNUITY

Many households would like the option to accept a lower lifetime annuity rather than an annuity that stops at life expectancy. Table A-1 continues the example of a male household and 2% inflation. The RSAM benefit in column I is the $1447 payment less a premium of $117. The balance in column II is greater after life expectancy (compared to Table 4-1) because payments continue; premiums to the lender continue as well. Returns in column VII are greater in the early years due to the lower net investment to the lender. The expected return for the IRRs in column VII is 10%. Equivalent premiums for female and couple households are $70 and $33 respectively.

Table A-1

RSAM with Lifetime Annuity

Age at Death	I RSAM Benefit	II Balance	III Appre-ciation	IV Total RSAM Balance	V Net Sale Price	VI Rever-sion	VII Return (IRR)
65	1330	1525	766	2291	36722	2291	.7223
66	1330	3131	1547	4678	37456	4678	.4410
67	1330	4824	2344	7168	38206	7168	.3231
68	1330	6607	3157	9764	38970	9764	.2583
69	1330	8486	3986	12472	39749	12472	.2175
70	1330	10465	4832	15297	40544	15297	.1895
71	1330	12551	5695	18246	41355	18246	.1690
72	1330	14749	6575	21323	42182	21323	.1534
73	1330	17064	7472	24536	43026	24536	.1412
74	1330	19503	8387	27891	43886	27891	.1313
75	1330	22073	9321	31394	44764	31394	.1232
76	1330	24781	10274	35055	45659	35055	.1164
77	1330	27634	11245	38879	46572	38879	.1106
78	1330	30640	12236	42876	47504	42876	.1057
79	1330	33807	13247	47054	48454	47054	.1014
80	1330	37144	14278	51422	49423	49423	.0934
81	1330	40659	15329	55989	50411	50411	.0839
82	1330	44364	16402	60765	51420	51420	.0758
83	1330	48266	17496	65762	52448	52448	.0687
84	1330	52378	18612	70990	53497	53497	.0627
85	1330	56710	19750	76460	54567	54567	.0573
86	1330	61274	20911	82185	55658	55658	.0527
87	1330	66084	22095	88179	56772	56772	.0486
88	1330	71150	23303	94453	57907	57907	.0450
89	1330	76489	24535	101024	59065	59065	.0417
90	1330	82113	25792	107905	60246	60246	.0389
91	1330	88039	27074	115113	61451	61451	.0363
92	1330	94283	28381	122664	62680	62680	.0340
93	1330	100861	29715	130576	63934	63934	.0319
94	1330	107792	31075	138867	65213	65213	.0300
95	1330	115094	32463	147557	66517	66517	.0283
96	1330	122788	33878	156666	67847	67847	.0268
97	1330	130894	35321	166216	69204	69204	.0254
98	1330	139435	36794	176229	70588	70588	.0242
99	1330	148433	38296	186729	72000	72000	.0230

BIOGRAPHICAL SKETCH

William H. Tate graduated from the United States Naval Academy in 1967 and received a Master of Science in Administration degree from George Washington University in 1973. He served as an instructor at the Naval Academy for three years and as an assistant professor at the Florida Institute of Technology for six years. Published works include a textbook and articles in the field of navigation. He holds the rank of Commander in the Naval Reserve.

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

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